KNT/KW/16/5139

Bachelor of Science (B.Sc.) Semester—IV (C.B.S.) Examination

MATHEMATICS (M_8-Mechanics)

Paper—II

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the **five** questions.

(2) All questions carry equal marks.

(3) Questions **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Prove that any system of coplanar forces acting at different points of rigid body can be reduced to a single force through a given point and a couple. 6

(B) Five weightless rods of equal length are joined together so as to form a rhombus ABCD with one diagonal BD. If a weight W is attached to C and the system is suspended from A, show that there is a thrust in BD equal to $\frac{W}{\sqrt{3}}$. 6

OR

(C) Derive the Cartesian equation of a common catenary in the form $y = c\cosh\left(\frac{x}{c}\right)$. 6

(D) If α and β be the inclinations to the horizon of the tangents at the extremities of a portion of a common catenary and *l* be the length of the portion, show that the height of one extremity over the other is $l \,.\, \sin\left(\frac{\alpha+\beta}{2}\right) / \cos\left(\frac{\alpha-\beta}{2}\right)$, the two extremities being on one side of the vertex of the catenary. 6

UNIT—II

2. (A) If the velocities of a particle along and perpendicular to a radius vector from a fixed origin are λr^2 and $\mu\theta^2$, show that the equation of the path is $\frac{\lambda}{\theta} = \frac{\mu}{2r} + c$ and the components of accelerations are $2\lambda^2 r^3 - \mu^2 \frac{\theta^4}{r}$ and $\lambda\mu r\theta^2 + 2\mu^2 \frac{\theta^3}{r}$. 6

(B) A particle is describing a plane curve. If the tangential and normal accelerations are constant throughout the motion, show that the angle Ψ, through which the direction of motion turns in time 't' is given by $\Psi = A \log(1 + Bt)$. 6

OR

(C) Show that a particle executing Simple Harmonic Motion requires $\frac{1}{6}$th of its period T, to move from the position of maximum displacement to one in which the displacement is half the amplitude. 6

(D) For a particle moving in Simple Harmonic Motion with amplitude 'a' and periodic time 'T', derive the expression of velocity 'ν' in terms of a, T and t and show that $\int_0^T \nu^2 dt = \frac{2\pi^2 a^2}{T}$. 6

UNIT—III

3. (A) Prove D' Alembert's principle that "the virtual work on a mechanical system by the applied forces and the reversed effective forces is zero" i.e.

$$\sum_i \left[\bar{F}_i^{(a)} - \dot{\bar{p}}_i\right] \circ \delta \bar{r}_i = 0, \quad i = 1, 2, \ldots.., n$$

where $\bar{F}_i^{(a)}$ is the applied force on the i^{th} particle of the system. 6

(B) Derive Lagrange's equations of motion in the form $\frac{d}{dt}\left(\frac{\partial L}{\partial q_j}\right) - \frac{\partial L}{\partial \dot{q}_j} = 0$, j = 1, 2, ..., n

for conservative system, where L = T – V is the Lagrangian of the system. 6

OR

(C) Define Rayleigh's dissipation function R, and show that the rate of energy dissipation due to friction is 2R. 6

(D) If L is the Lagrangian for a system having n degrees of freedom, show that $L' = L + \frac{dF}{dt}$ also satisfies Lagrange's equation, where $F = F(q_1, q_2, \ldots, q_n, t)$ is any arbitrary differentiable function. 6

UNIT—IV

4. (A) Prove that the problem of motion of two masses interacting only with each other can always be reduced to a problem of motion of a single mass. 6

(B) For a system moving in a finite region of space with finite velocity, prove that the time average of the kinetic energy is equal to the virial of the system i.e. $\overline{T} = \overline{-\frac{1}{2}\sum_i \overline{F}_i \circ \overline{r}_i}$. 6

OR

(C) For a central force field F, derive the path of a particle of mass 'm' in the form :

$$\frac{d^2u}{d\theta^2} + u = -\frac{m}{h^2u^2} F\left(\frac{1}{u}\right), \text{ where } u = \frac{1}{r}.$$ 6

(D) A particle moves on a curve $r^n = a^n \cos n\theta$ under the influence of a central force field. Prove that $f(r) \propto r^{-(2n+3)}$. 6

Question—V

5. (A) Define virtual work done and state the principle of virtual work. 1½

(B) Show that in a catenary, $s = c \sinh \frac{x}{c}$. 1½

(C) Find the normal acceleration of a particle describing a cycloid $s = 4a \sin \psi$. 1½

(D) Show that if the displacement of a particle is $x = a \cos nt + b \sin nt$, then it executes Simple Harmonic Motion. 1½

(E) Write the Lagrangian of a particle moving in plane, in polar coordinates. 1½

(F) Define velocity dependent potential. 1½

(G) Show that the path of a particle in a central force field lies in one plane. 1½

(H) If the conservative force F is given by $F = \frac{-k}{r^2}$, find the potential V. 1½